

An Occupation-Time Approach to Relative Drawdown

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We study a stochastic control problem motivated by portfolio management under relative performance evaluation. An investor's wealth process is compared against a benchmark - an exogenous, non-replicable reference process (e.g., a stock market index) - and the quantity of interest is the benchmark-relative log-drawdown, the shortfall of the (log) wealth-to-benchmark ratio below its running maximum. Rather than penalizing the magnitude of this drawdown, we penalize its *persistence*: the objective is the expected discounted occupation time that the drawdown process spends beyond a fixed critical threshold.

Although the drawdown introduces path dependence through its running-maximum term, we show that the problem reduces to a one-dimensional Markovian control problem and derive the corresponding Hamilton-Jacobi-Bellman equation. The optimal feedback control admits an explicit characterization as a metric projection of a state-dependent vector onto a convex constraint set. We prove a verification theorem and give geometric conditions under which the resulting reflected diffusion possesses a unique strong solution.

References:

- [1] J. Sekine, M. Wunsch: Minimizing Benchmark-Relative Drawdown Duration via Occupation Time Penalization, *arXiv:2607.11335 [q-fin.MF]*