

Beyond the positive drift: Comparing historical and current daily temperature patterns based on two sample statistics for unbalanced dense-sparse functional data

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The two-sample problem for functional data is investigated for discrete, synchronous designs in each sample, in settings in which one sample is densely observed while the other is only relatively sparsely observed. This is motivated by comparing historical and more current daily temperature patterns, where more recent devices take measurements every 10 minutes, while historical measurements in the time period 1952 to 1972 are available only every hour. We use recently developed methods from transfer learning for functional data to estimate the difference of the mean functions at optimal rates in the supremum norm. Further, we derive a central limit theorem in the space of continuous functions and discuss the construction of uniform confidence bands using the multiplier bootstrap. We also show how our methods can be extended to functional time series. In the application to daily temperature patterns we decompose the mean difference function into a daily average - the normalized integral of the mean difference function - as well as into the deviation from the average value. Using the developed inferential methodology we show that not only the daily average temperatures in each month have increased significantly, but also that the daily temperature patterns have changed for most months, with night temperature remaining relatively stable while daily temperatures increasing beyond the daily average value.