Optimal anticoncentration of spanning trees in pseudorandom graphs

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Let G be an n-vertex d-regular graph. Lee established the following anticoncentration inequality for spanning trees: Let $\mathbf{T}$ be an uniformly random spanning tree in T and T be an arbitrary spanning tree on n vertices. Then, $\mathbb{P}(T \simeq \mathbf{T}) \leq \exp(-n/2000)$. In particular, it implies there are $\exp(n/2000)$ non-isomorphic spanning trees in G.
Our main result gives the asymptotically optimal anticoncentration bound under additional spectral constraints: For every $\varepsilon > 0$, there exists $C > 0$ such that if G is an (n, d, λ)-graph with $d/\lambda \geq C$, then

$$\mathbb{P}(T \simeq \mathbf{T}) \leq \exp(-(1-\varepsilon)n)\,.$$

This resolves an open problem from Lee's paper and the bound is asymptotically optimal.

References:

[1] H. Lee: Anticoncentration of random spanning trees in almost regular graphs, *arXiv: 2601.07740v1*, 2026.