Supercritical sharpness of percolation beyond $\mathbb{Z}^d$

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Percolation provides one of the simplest mathematical models of a phase transition: edges of a network are randomly opened, and beyond a critical probability an infinite connected component appears. Understanding how rapidly the system changes as this threshold is crossed is a fundamental problem in probability.

This supercritical behavior was understood long ago for Euclidean lattices, and more recently for several other important classes of graphs, including graphs of polynomial growth and non-amenable graphs. Extending it to full generality, however, has proved much more difficult. We establish supercritical sharpness for every infinite transitive graph, showing that above the critical threshold large finite components are exponentially rare, with the correct rate determined by the geometry of the underlying graph. I will discuss the problem in more detail and explain the main ideas behind the proof.

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