

Talagrand-type transport inequalities for path spaces over Carnot groups

Peter K. Friz^{a,b}, Helena Kremp^{a,b}, Vaios Laschos^c, Matthias Liero^b, Benjamin A. Robinson^d

^a Technische Universität Berlin, Berlin, Germany.

^b Weierstraß-Institut für Angewandte Analysis und Stochastik, Berlin, Germany.

^c kausable GmbH, Heidelberg, Germany.

^d Department of Statistics, University of Klagenfurt, Austria.

Talagrand's transportation inequality relates the Wasserstein distance and relative entropy, and is known to hold, for example, when the reference measure is a standard Gaussian or the law of a Brownian motion on Euclidean space.

We consider Talagrand-type transportation inequalities for the law of Brownian motion on Carnot groups. An important example is the lift of standard Brownian motion to the Brownian rough path. We present a direct proof on enhanced path space, which also yields equality when restricting to adapted couplings in the transport problem. Moreover, we prove a Talagrand inequality for the heat kernel measure on Carnot groups and deduce the inequality for the law of Brownian motion on Carnot groups via a bottom-up argument.

With a non-commutative sub-Riemannian state space, we observe phenomena that differ from the Euclidean case. In particular, while a top-down projection argument recovers Talagrand's inequality on Euclidean space from the corresponding inequality on the path space, such a projection argument breaks down in the Carnot group setting.

References:

- [1] Peter K. Friz, Helena Kremp, Vaios Laschos, Matthias Liero, and Benjamin A. Robinson: Talagrand-type transport inequalities for path spaces over Carnot groups, *arXiv:2602.06646*, 2026.