Nonnegative estimation of functions of probabilities

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Statistical procedures for intractable models often require unbiased estimation of a quantity $f(x)$, where x is unknown but simple to estimate and f is known but nonlinear. Examples include MCMC for big data, where $f(\cdot) = \exp(\cdot)$ and x is a log-likelihood estimator obtained from a mini-batch, and for doubly intractable targets where $f(x) = 1/x$ and x is a normalising constant for the likelihood. Consistent (but not unbiased) estimation of $f(x)$ is straightforward in each case, and a consistent sequence of estimators can be converted into an unbiased estimator via random truncation of a telescoping series known as debiasing [2]. However, the resulting unbiased estimators are not nonnegative in general, and hence cannot immediately be used in MCMC applications. It is known that unbiased, nonnegative estimators for general functions f do not exist [1]. I will show that when $f : [0, 1] \to (0, 1)$ and $p \in [0, 1]$, choosing the sequence of consistent estimators based on the coefficients of a particular Bernoulli factory produces an unbiased, $[0, 1]$-valued estimator of $f(p)$ under relatively mild smoothness assumptions. While the $[0, 1]$-valued estimator is straightforward to implement only in trivial examples, I will also demonstrate that an implementable, unbiased, and nonnegative estimator of $f(p)$ is obtainable as an easy corollary.

References:

[1] P. E. Jacob and A. H. Thiery: On nonnegative unbiased estimators, *Annals of Statistics*, 43 (2015), 769–784.

[2] D. McLeish: A general method for debiasing a Monte Carlo estimator, *Monte Carlo Methods and Applications*, 17 (2011), 301–315.