Two-sided heat kernel bounds for $\sqrt{8/3}$-Liouville Brownian motion

N. Kajino[a], *joint work with S. Andres*[b], *K. Kavvadias*[c], *and J. Miller*[d]

[a] Kyoto University.

[b] Technische Universität Braunschweig.

[c] Massachusetts Institute of Technology.

[d] University of Cambridge.

The γ-*Liouville Brownian motion* (γ-*LBM*) is the canonical diffusion process on a γ-Liouville quantum gravity (γ-LQG) surface, where $\gamma \in (0, 2)$ is a parameter. The γ_0-LQG sphere considered as (the isomorphism class of) a random compact metric measure space, where $\gamma_0 := \sqrt{8/3}$, has been proved in [5, 6, 7] to have the same law as the Brownian sphere, which in turn was proved in [2, 4] (see also [3, Theorem 7]) to appear as the scaling limit of a uniform random n-face quadangulation of the sphere as $n \to \infty$.
This talk will present the main result of [1] establishing, for the heat kernel $p_t^{\gamma_0}(x, y)$ of the γ_0-LBM on the γ_0-LQG sphere $(\mathbb{S}^2, \rho_{\gamma_0}, \mu_{\gamma_0})$, the following two-sided off-diagonal sub-Gaussian bounds, which are sharp up to polylogarithmic factors in the exponential.

Theorem ([1, Theorems 1.1 and 1.2]). There exist a non-random constant $\kappa \in (0, \infty)$ and random constants $C_1, C_2, C_3, C_4 \in (0, \infty)$ such that for almost-every instance of the γ_0-LQG sphere $(\mathbb{S}^2, \rho_{\gamma_0}, \mu_{\gamma_0})$,

$$\begin{aligned}
p_t^{\gamma_0}(x, y) &\leq C_1 t^{-1} \bigl(\log(t^{-1})\bigr)^{\kappa} \exp\bigl(-C_2 \Phi_{-\kappa}(\rho_{\gamma_0}(x, y), t)\bigr), \\
p_t^{\gamma_0}(x, y) &\geq \exp\bigl(-\Phi_{\kappa}(\rho_{\gamma_0}(x, y), t)\bigr) \mathbf{1}_{[C_3,\infty)}(\rho_{\gamma_0}(x, y)/t), \\
p_t^{\gamma_0}(x, x) &\geq C_4 t^{-1} \bigl(\log(t^{-1})\bigr)^{-\kappa}
\end{aligned}$$

for any $(t, x, y) \in (0, \frac{1}{2}] \times \mathbb{S}^2 \times \mathbb{S}^2$, where $\Phi_{\pm\kappa}(\rho, t) := (\rho^4/t)^{1/3} \bigl(\log(e + \rho/t)\bigr)^{\pm\kappa}$ for $(\rho, t) \in [0, \infty) \times (0, \infty)$.

References:

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