Algebraic aspects of the polynomial Littlewood–Offord problem

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Consider a degree-d polynomial $f(\xi_1, \ldots, \xi_n)$ of independent Rademacher random variables $\xi_1, \ldots, \xi_n$. To what extent can $f(\xi_1, \ldots, \xi_n)$ concentrate on a single point? This is the so-called *polynomial Littlewood–Offord problem*. A nearly optimal bound was proved by Meka, Nguyen and Vu: the point probabilities are always at most about $1/\sqrt{n}$, unless f is "close to the zero polynomial" (having only $o(n^d)$ nonzero coefficients).
In this project, we establish several results supporting the general philosophy that the Meka–Nguyen–Vu bound can be significantly improved unless f is "close to a polynomial with special algebraic structure", drawing some comparisons to phenomena in analytic number theory. In particular, one of our results is a corrected version of a conjecture of Costello on multilinear forms.

References:

[1] Z. Jin, M. Kwan, L. Sauermann, and Y. Wang: Algebraic aspects of the polynomial Littlewood–Offord problem, arXiv preprint arXiv:2505.23335 (2025).