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Contents

Random Matrices, Intrinsic Freeness, and Sharp Non-Asymptotic Inequalities

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Random matrix theory has played a major role in several areas of pure and applied mathematics, as well as statistics, physics, and computer science. This lecture aims to describe the intrinsic freeness phenomenon and how it provides new easy-to-use sharp non-asymptotic bounds on the spectrum of general random matrices.

Splitting methods for stochastic Hodgkin-Huxley type systems

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In this talk, we consider a class of locally Lipschitz SDEs of Hodgkin-Huxley type, characterized by a conditionally linear drift structure. For these systems, we construct different Lie-Trotter and Strang splitting methods exploiting their conditional linearity. In contrast to Euler-Maruyama type methods, the proposed splitting schemes are inherently structure-preserving. In particular, we prove that they are geometrically ergodic and preserve the natural state space of the model. Numerical experiments demonstrate that the splitting schemes are markedly superior in preserving the qualitative dynamics of neuronal spiking and allow for significantly larger time steps.

While structure preservation follows naturally from the construction of the splitting methods, establishing mean-square convergence is considerably more challenging. Existing fundamental theorems for mean-square convergence of numerical methods for SDEs require globally or one-sided Lipschitz continuous coefficients, while strong convergence results under merely local Lipschitz conditions are largely restricted to Euler-Maruyama type methods. To address these limitations, we introduce a new localized version of the fundamental mean-square convergence theorem for SDEs with locally Lipschitz coefficients. Specifically, we show that if a numerical scheme is locally consistent in the mean-square sense of order $q > 1$, then it is locally mean-square convergent with rate $q - 1$. Building on this result, we further prove that global mean-square convergence (without rate) follows, provided that both the exact solution and its numerical approximation admit bounded moments. This convergence framework is then applied to the splitting schemes, requiring innovative proofs of local consistency and boundedness of moments.

References:

[1] P. Étoré, A. Melnykova and I. Tubikanec: Splitting methods for stochastic Hodgkin-Huxley type systems and a localized fundamental mean-square convergence theorem, *arXiv preprint arXiv:2602.13056*, 2026.

Supercritical sharpness of percolation beyond $\mathbb{Z}^d$

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Percolation provides one of the simplest mathematical models of a phase transition: edges of a network are randomly opened, and beyond a critical probability an infinite connected component appears. Understanding how rapidly the system changes as this threshold is crossed is a fundamental problem in probability.

This supercritical behavior was understood long ago for Euclidean lattices, and more recently for several other important classes of graphs, including graphs of polynomial growth and non-amenable graphs. Extending it to full generality, however, has proved much more difficult. We establish supercritical sharpness for every infinite transitive graph, showing that above the critical threshold large finite components are exponentially rare, with the correct rate determined by the geometry of the underlying graph. I will discuss the problem in more detail and explain the main ideas behind the proof.

Joint work with Sahar Diskin, Philip Easo, Ritvik Ramanan Radhakrishnan, and Vincent Tassion.

Persistence of crossings in dynamical planar percolation

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In the $n \times n$ rhombus on the triangular lattice, let each vertex switch between open and closed according to iid Poisson clocks of rate 1. At any moment, this is critical site percolation, with the probability of a left-right open crossing being exactly 1/2. The existence of a crossing is extremely noise-sensitive: in the dynamics, it decorrelates on a time scale $\rho(n) = n^{-3/4+o(1)}$, which is much shorter than the constant relaxation time of the entire system.

What is the probability that a crossing exists continually for time $t\rho(n)$, large t? Hammond-Pete-Mossel (2012) proved that this is at least exponential, but decays faster than any polynomial in t. My current joint work with Malo Hillairet gives a lower bound $\exp(-t^{2/3+o(1)})$, which might actually be the true behaviour. The proof makes sense of a naive renormalization heuristic proposed 15 years ago.

A one–stable phase transition in Gibbs partitions

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We study critical Gibbs partitions associated with composition schemes, in the borderline case where the inner weights decay as n^{-2} up to a slowly varying factor. The corresponding component-size distribution belongs to the domain of attraction of an asymmetric stable law. When the outer weights of the composition have exponent in $(1, 2)$, we establish a dense phase in which the number of components exhibits Cauchy fluctuations around an explicit deterministic scale. We also obtain a functional one-stable limit theorem and describe the asymptotic distribution of the largest components through an appropriate point process. We also identify a phase transition between the dense regime and a condensation regime featuring a giant component. Depending on the relative strength of the two contributions, the limiting model is purely dense, purely condensed, or a non-trivial mixture of both phases.

Some recent results on the Stochastic Sandpile Model

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The Stochastic Sandpile Model is an interacting particle system introduced in the physics literature in the '90s to study the concept of self-organized criticality, which describes physical systems that spontaneously evolve toward a critical state without the need to fine-tune their parameters. In this talk, I will present the model and discuss some questions related to its behaviour, such as the critical density at which the phase transition occurs, how to exactly sample from the stationary distribution on finite graphs, and the stationary particle density on the complete graph.

K-branching random walk with noisy selection: large population limits and phase transitions

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Branching particle systems with selection have recently received significant attention as models for understanding the interplay between reproduction, competition, and evolutionary dynamics. In this talk, I will present results on a K-particle branching random walk with noisy selection. At each generation, K particles lie on the real line with their position representing fitness. The particles first undergo a reproduction phase, in which each particle gives birth to a fixed number of children whose positions are independent and identically distributed random displacements from their parent's position. This is followed by a noisy selection phase whereby K particles are randomly selected from the offspring population with selection probabilities favouring fitter individuals.

For a general class of reproduction distributions, we provide an asymptotic description of how the population distribution evolves from one generation to the next. For displacement probabilities with exponential tails we prove the existence of asymptotic travelling wave solutions for the population distributions. These solutions undergoees a phase transition as the selection pressures vary. In what we call the *survival of the fittest* regime, with high probability all of the fittest individuals past a certain threshold are selected. In the other regime, with high probability none of the fittest individuals past a certain threshold are selected, what we call *survival of the luckiest.*

This is joint work with Emmanuel Schertzer and Zsófia Talyigás, and is available on arXiv [1].

References:

[1] C. Desmarais, E. Schertzer, and Z. Talyigás, K-Branching Random Walk with Noisy Selection: Large Population Limits and Phase Transitions, *available online at* `https://arxiv.org/abs/2509.26254`, (2025).

Machine Learning Methods in Portfolio Optimisation

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This study investigates the use of machine learning and reinforcement learning methods to determine optimal investment strategies that maximise the expected utility of terminal wealth. The proposed framework connects classical stochastic control methods with computational learning approaches in both one-dimensional and two-dimensional financial market models.

Portfolio optimisation problems are formulated within the binomial market model and the two-dimensional Rendleman-Bartter model [2]. Solution techniques, including the martingale optimality principle [1], backward induction, forward induction, and dynamic programming, provide the theoretical foundation. Reinforcement learning methods, particularly Q-learning [3] and Q-learning with a neural network, are then applied to approximate optimal investment decisions. In the Q-learning approach with a neural network, a radial basis function neural network is used to approximate the Q-function and to represent the relationship between wealth states, investment actions, and expected terminal utility.

The framework is further extended to include transaction costs in the binomial market model, allowing investment decisions to be examined under more realistic market conditions. The numerical results illustrate how reinforcement learning methods can approximate the optimal strategies obtained using classical stochastic control approaches.

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Viability problems for SDEs in a general Gelfand triple setup using a variational approach

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We investigate some viability conditions for a general class of stochastic differential equations, studied in the variational approach:

$$dX_t = A(X_t)dt + B(X_t)dW_t.$$

The space setup that we consider is given by a general Gelfand triple $V \subset H \subset V^*$, where H is a separable Hilbert space, on which we should understand our equation. We provide necessary and sufficient characterization for the viability of closed random constraints in terms of adapted variational tangent or contingent sets. Several important classes of examples which can be considered are also envisaged.

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Martingale Solutions for 3D stochastic Chemotaxis-fluid model

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In this talk, we present an existence result for a stochastic model describing the collective dynamics of oxygen-driven swimming bacteria in an aquatic fluid flowing through a bounded three-dimensional domain and subject to random external forces. The model consists of the stochastic Navier–Stokes equations coupled with a stochastic Keller–Segel system. Our analysis establishes the existence of martingale solutions to this coupled stochastic system, providing a mathematical framework for describing the interplay between bacterial chemotaxis, fluid motion, and environmental randomness.

Algebraic aspects of the polynomial Littlewood–Offord problem

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Consider a degree-d polynomial $f(\xi_1, \ldots, \xi_n)$ of independent Rademacher random variables $\xi_1, \ldots, \xi_n$. To what extent can $f(\xi_1, \ldots, \xi_n)$ concentrate on a single point? This is the so-called *polynomial Littlewood–Offord problem*. A nearly optimal bound was proved by Meka, Nguyen and Vu: the point probabilities are always at most about $1/\sqrt{n}$, unless f is "close to the zero polynomial" (having only $o(n^d)$ nonzero coefficients).

In this project, we establish several results supporting the general philosophy that the Meka–Nguyen–Vu bound can be significantly improved unless f is "close to a polynomial with special algebraic structure", drawing some comparisons to phenomena in analytic number theory. In particular, one of our results is a corrected version of a conjecture of Costello on multilinear forms.

References:

[1] Z. Jin, M. Kwan, L. Sauermann, and Y. Wang: Algebraic aspects of the polynomial Littlewood–Offord problem, arXiv preprint arXiv:2505.23335 (2025).

Stochastic Sandpiles with Uniform Topplings on the Line

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We discuss the stochastic sandpile model with uniform topplings on the line: vertices are declared unstable if 2 or more particles are placed on them, and unstable vertices are allowed to legally topple. During a toppling, with probability 1/3 a particle is sent to right, with probability 1/3 a particle is sent to the left and with probability 1/3 two particles are sent out, one to the left and one to the right. Using these dynamics, we can define the stochastic sandpile Markov chain on finite line segments, where in each time step we add a particle at a vertex chosen uniformly at random, and then we stabilize the configuration by performing all possible legal topplings. Particles sent outside the finite line segment are lost. During my talk, I will discuss results on the stationary distribution of this Markov chain, including an exact expression of the stationary distribution and its infinite volume limit.

Two-sided heat kernel bounds for $\sqrt{8/3}$-Liouville Brownian motion

N. Kajino[a], joint work with S. Andres[b], K. Kavvadias[c], and J. Miller[d]

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The γ*-Liouville Brownian motion (γ-LBM)* is the canonical diffusion process on a γ-Liouville quantum gravity (γ-LQG) surface, where $\gamma \in (0,2)$ is a parameter. The γ_0-LQG sphere considered as (the isomorphism class of) a random compact metric measure space, where $\gamma_0 := \sqrt{8/3}$, has been proved in [5, 6, 7] to have the same law as the Brownian sphere, which in turn was proved in [2, 4] (see also [3, Theorem 7]) to appear as the scaling limit of a uniform random n-face quadangulation of the sphere as $n \to \infty$.

This talk will present the main result of [1] establishing, for the heat kernel $p_t^{\gamma_0}(x,y)$ of the γ_0-LBM on the γ_0-LQG sphere $(\mathbb{S}^2, \rho_{\gamma_0}, \mu_{\gamma_0})$, the following two-sided off-diagonal sub-Gaussian bounds, which are sharp up to polylogarithmic factors in the exponential.

Theorem ([1, Theorems 1.1 and 1.2]). There exist a non-random constant $\kappa \in (0,\infty)$ and random constants $C_1, C_2, C_3, C_4 \in (0,\infty)$ such that for almost-every instance of the γ_0-LQG sphere $(\mathbb{S}^2, \rho_{\gamma_0}, \mu_{\gamma_0})$,

$$\begin{aligned}
p_t^{\gamma_0}(x,y) &\le C_1 t^{-1}\bigl(\log(t^{-1})\bigr)^{\kappa} \exp\bigl(-C_2 \Phi_{-\kappa}(\rho_{\gamma_0}(x,y),t)\bigr),\\
p_t^{\gamma_0}(x,y) &\ge \exp\bigl(-\Phi_{\kappa}(\rho_{\gamma_0}(x,y),t)\bigr)\mathbf{1}_{[C_3,\infty)}(\rho_{\gamma_0}(x,y)/t),\\
p_t^{\gamma_0}(x,x) &\ge C_4 t^{-1}\bigl(\log(t^{-1})\bigr)^{-\kappa}
\end{aligned}$$

for any $(t,x,y) \in (0,\frac{1}{2}] \times \mathbb{S}^2 \times \mathbb{S}^2$, where $\Phi_{\pm\kappa}(\rho,t) := (\rho^4/t)^{1/3}\bigl(\log(e+\rho/t)\bigr)^{\pm\kappa}$ for $(\rho,t) \in [0,\infty) \times (0,\infty)$.

References:

[1] S. Andres, N. Kajino, K. Kavvadias and J. Miller, *Two-sided heat kernel bounds for $\sqrt{8/3}$-Liouville Brownian motion*, preprint, 2025. `https://doi.org/10.48550/arXiv.2507.13269`

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Precise Large Deviations of the Ewens-Pitman Model

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This work is joint with Thomas Lehéricy, Pierre-Loic Méliot and Ashkan Nikeghbali.

The Ewens-Pitman model was introduced in [1] by Pitman as a generalization of the Ewens model in population genetics ([2]). For any $n \in \mathbb{N}$, we consider a random partition of $[n] = \{1, \dots, n\}$ into $K_n \in \{1, \dots, n\}$ partition sets, and $K_{r,n} \in \{0, 1, \dots, n\}$ is the number of partition subsets of size r, for any $r = 1, \dots, n$:

$$n = \sum_{r=1}^{n} r K_{r,n}, \quad K_n = \sum_{r=1}^{n} K_{r,n}.$$

For $\alpha \in [0, 1)$ and $\theta > -\alpha$, the Ewens-Pitman model assigns to $\mathbf{K}_n = (K_{1,n}, \dots, K_{n,n})$ the probability

$$\mathbb{P}_{\alpha,\theta}(\mathbf{K}_n = (k_1, \dots, k_n)) = n! \frac{\left(\frac{\theta}{\alpha}\right)^{\left(\sum_{i=1}^{n} k_i\right)}}{(\theta)^{(n)}} \prod_{i=1}^{n} \left(\frac{\alpha(1-\alpha)^{(i-1)}}{i!} \right)^{k_i} \frac{1}{k_i!},$$

where $(a)^{(m)} = a(a+1) \cdots (a+m-1)$ for all $a \in \mathbb{R}$ and $m \in \mathbb{N} \cup \{0\}$ ($(a)^{(0)} = 1$). Setting $\alpha = 0$ corresponds to the Ewens model.

There is a significant number of works about asymptotic behaviour of the number K_n of partition sets in the Ewens-Pitman model. Now, we are interested in large deviations. For $\alpha = 0$, K_n is a sum of independent Bernoulli random variables ([3]), from which it follows easily that (K_n/n) satisfies a large deviation principle with good rate function

$$I_\theta(x) = \begin{cases} x \ln\left(\frac{x}{\theta}\right) - x + \theta, & x > 0, \\ \theta, & x = 0, \\ +\infty, & x < 0. \end{cases}$$

For $\alpha \in (0, 1)$, this is not true. However, in [4], Feng and Hoppe proved large deviation principle with good rate function $I_\alpha(x) = \sup_{t \in \mathbb{R}}(xt - L_\alpha(t))$, where

$$L_\alpha(t) = \begin{cases} -\ln\left(1 - (1 - e^{-t})^{1/\alpha}\right) & t > 0, \\ 0, & t \leq 0. \end{cases}$$

We obtain precise large deviation principle for (K_n/n) with $\alpha \in (0, 1)$, considering the moment-generating function of K_n with using some results of Bercu and Favaro ([5]). Our research is based on improving Chaganty-Sethuraman's result ([6]) about local limit theorems and precise large deviations for the lattice case that has been recently obtained by us with using the approach suggested in [7] by Delbaen, Kowalski, and Nikeghbali. Notice that, in this case, we do not have mod-φ convergence and we cannot apply some already known results of Large Deviation theory. Thus, one of the most important parts of our work is to weaken assumptions on the known general results and to extend their applications.

It is worth mentioning that, in [8], Peng and Zhou recently proved precise large deviation principle for the Ewens-Pitman model, but their approach is different from ours and does not allow us to clearly see the connection with Feng-Hoppe large deviation principle.

References:

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The eigenvalues of I.I.D. matrices are hyperuniform

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We consider the point process of the eigenvalues of real or complex non-Hermitian matrices X with independent, identically distributed entries. In this talk, I will present a recent result obtained in a joint work with L. Erdos and G. Cipolloni [1], which establishes that this point process is hyperuniform: the variance of the number of eigenvalues in a subdomain of the spectrum is much smaller than the volume of this subdomain. Our main technical novelty is a very precise computation of the covariance between the resolvents of the Hermitization of $X - z_1$, $X - z_2$, for two distinct complex parameters z_1, z_2.

References:

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Nonnegative estimation of functions of probabilities

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Statistical procedures for intractable models often require unbiased estimation of a quantity $f(x)$, where x is unknown but simple to estimate and f is known but nonlinear. Examples include MCMC for big data, where $f(\cdot) = \exp(\cdot)$ and x is a log-likelihood estimator obtained from a mini-batch, and for doubly intractable targets where $f(x) = 1/x$ and x is a normalising constant for the likelihood. Consistent (but not unbiased) estimation of $f(x)$ is straightforward in each case, and a consistent sequence of estimators can be converted into an unbiased estimator via random truncation of a telescoping series known as debiasing [2]. However, the resulting unbiased estimators are not nonnegative in general, and hence cannot immediately be used in MCMC applications. It is known that unbiased, nonnegative estimators for general functions f do not exist [1]. I will show that when $f : [0,1] \to (0,1)$ and $p \in [0,1]$, choosing the sequence of consistent estimators based on the coefficients of a particular Bernoulli factory produces an unbiased, $[0,1]$-valued estimator of $f(p)$ under relatively mild smoothness assumptions. While the $[0,1]$-valued estimator is straightforward to implement only in trivial examples, I will also demonstrate that an implementable, unbiased, and nonnegative estimator of $f(p)$ is obtainable as an easy corollary.

References:

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A stochastic Schauder-Tychonoff type theorem and its applications

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One standard way to prove existence for deterministic, highly nonlinear PDEs is to use the Schauder–Tychonoff fixed-point theorem. In what follows, we introduce and verify a stochastic variant of the Schauder–Tychonoff theorem. We apply our existence result to nonlinear stochastic diffusion equations with non-Lipschitz perturbations.

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Deterministic and Monte Carlo Approximation scheme for Stochastic Differential Equations with Interaction

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We consider the stochastic differential equation with interaction introduced by A. A. Dorogovtsev [1, 2]:

$$\begin{aligned} dx(u,t) &= a(x(u,t),\mu_t)\,dt + \int_{\mathbb{R}^d} b(x(u,t),\mu_t,p)\,W(dp,dt), \\ x(u,0) &= u, \qquad u \in \mathbb{R}^d, \qquad \mu_t = \mu_0 \circ x(\cdot,t)^{-1}. \end{aligned} \tag{1}$$

Here W is a Wiener sheet on $\mathbb{R}^d \times [0,+\infty)$,

$$a : \mathbb{R}^d \times M_2(\mathbb{R}^d) \to \mathbb{R}^d, \qquad b : \mathbb{R}^d \times M_2(\mathbb{R}^d) \times \mathbb{R}^d \to \mathbb{R}^{d\times d},$$

and $p \mapsto b(x,\mu,p)$ belongs to $L_2(\mathbb{R}^d;\mathbb{R}^{d\times d})$ for every (x,μ). For each u, the process $x(u,\cdot)$ is the trajectory of the particle starting from u, while μ_t is the image of the initial mass distribution under the random map $u \mapsto x(u,t)$. Thus, the coefficients at time t depend on both the position of an individual particle and the current mass distribution of the entire system.

For $p \geq 1$, let $M_p(\mathbb{R}^d)$ denote the space of all Borel probability measures μ on $\mathbb{R}^d$ with finite pth moment,

$$\int_{\mathbb{R}^d} |z|^p\,\mu(dz) < \infty.$$

For $\mu,\nu \in M_p(\mathbb{R}^d)$, the Wasserstein distance of order p is defined by

$$\gamma_p(\mu,\nu) := \left(\inf_{\varkappa \in C(\mu,\nu)} \int_{\mathbb{R}^d\times\mathbb{R}^d} |x-y|^p\,\varkappa(dx,dy) \right)^{1/p}, \tag{2}$$

where $C(\mu,\nu)$ is the set of all couplings of μ and ν [1, 2].

We impose the following global Lipschitz assumptions: there exists $L > 0$ such that, for all $x,y \in \mathbb{R}^d$ and $\mu,\nu \in M_2(\mathbb{R}^d)$,

$$|a(x,\mu) - a(y,\nu)| \leq L\big(|x-y| + \gamma_2(\mu,\nu)\big), \tag{3}$$

$$\int_{\mathbb{R}^d} |b(x,\mu,p) - b(y,\nu,p)|^2\,dp \leq L^2\big(|x-y|^2 + \gamma_2^2(\mu,\nu)\big). \tag{4}$$

Under these assumptions, the stochastic differential equation with interaction admits a unique solution on $[0,+\infty)$. Moreover, for every $t > 0$, the measure $\mu_t = \mu_0 \circ x(\cdot,t)^{-1}$ is a random element in $\mathfrak{M}_2(\mathbb{R}^d)$ [2].

Let's consider the following examples of equations with interaction:

Kuramoto system [8]

Let

$$\mathbb{T} := \mathbb{R}/(2\pi\mathbb{Z})$$

be the one-dimensional torus. Since each oscillator is characterized by its phase $\vartheta_i(t) \in \mathbb{T}$ and its fixed frequency $\omega_i \in \mathbb{R}$, we consider the state space

$$E := \mathbb{T} \times \mathbb{R}, \qquad z_i(t) := (\vartheta_i(t), \omega_i).$$

Define the empirical measure

$$\mu_t^N := \frac{1}{N} \sum_{j=1}^{N} \delta_{z_j(t)} = \frac{1}{N} \sum_{j=1}^{N} \delta_{(\vartheta_j(t), \omega_j)}. \tag{5}$$

The Kuramoto system in integral form is

$$\vartheta_i(t) = \vartheta_i(0) + \int_0^t \left[\omega_i + \frac{K}{N} \sum_{j=1}^{N} \sin(\vartheta_j(s) - \vartheta_i(s)) \right] ds. \tag{6}$$

Equivalently, it can be written as an equation with interaction

$$z_i(t) = z_i(0) + \int_0^t A(z_i(s), \mu_s^N)\, ds, \tag{7}$$

where

$$A((\vartheta, \omega), \mu) := \left(\omega + K \int_E \sin(\vartheta' - \vartheta)\, \mu(d\vartheta', d\omega'),\, 0 \right). \tag{8}$$

Helmholtz point-vortex system [9]

For vortex positions $r_i(t) \in \mathbb{R}^2$ and intensities k_i, The integral form of the particle system is

$$r_i(t) = r_i(0) - \frac{1}{\pi} \int_0^t \sum_{j \neq i} k_j \nabla \log |r_i(s) - r_j(s)|\, ds. \tag{9}$$

Equivalently,

$$r_i(t) = r_i(0) + \int_0^t A(r_i(s), \mu_s)\, ds, \tag{10}$$

with

$$A(v, \mu) := -\frac{1}{\pi} \int_{\mathbb{R}^2} \mathbf{1}_{\{y \neq v\}} \nabla \log |v - y|\, \mu(dy). \tag{11}$$

Equation (10) has the measure-dependent structure of an equation with interaction.

Vortex filament and Landau–Lifshitz equations [10]

Let $\Gamma = \Gamma(s,t) \in \mathbb{R}^3$ be a filament parametrized by arc length, $|\partial_s \Gamma(s,t)| = 1$. The vortex-filament equation is

$$\Gamma(s,t) = \Gamma(s,0) + \int_0^t \partial_s \Gamma(s,\tau) \times \partial_{ss} \Gamma(s,\tau)\, d\tau. \tag{12}$$

Equivalently, since $\partial_s \Gamma \times \partial_{ss} \Gamma = \kappa B$, one has $\partial_t \Gamma = \kappa B$, where κ and B are the curvature and the binormal vector.

If

$$T(s,t) := \partial_s \Gamma(s,t), \qquad |T(s,t)| = 1, \tag{13}$$

then differentiation of (12) with respect to s gives the isotropic Landau–Lifshitz equation

$$T(s,t) = T(s,0) + \int_0^t T(s,\tau) \times \partial_{ss} T(s,\tau)\, d\tau. \tag{14}$$

[6, 7].

A natural approximation strategy for equation (1) is to discretize both the spatial dependence of the initial measure and the time variable. For compactly supported initial distributions, this can be achieved by replacing μ_0 with a finitely supported measure on a regular spatial grid and then applying the Euler–Maruyama scheme in time. However, the number of grid nodes grows rapidly with the dimension of the state space. This motivates replacing the deterministic spatial discretization by an empirical Monte Carlo measure, which requires only N sampled support points and remains applicable to initial distributions with unbounded support under suitable moment assumptions.

1. Deterministic spatial and temporal approximation. Assume

$$\operatorname{supp}\mu_0 \subset K = [-R, R]^d. \tag{15}$$

For $N \in \mathbb{N}$, let

$$h_N := \frac{2R}{N}$$

and introduce the regular Cartesian grid

$$\mathcal{G}_N := \left\{ u_{\mathbf{i}}^N = (-R + i_1 h_N, \ldots, -R + i_d h_N) : \mathbf{i} = (i_1, \ldots, i_d) \in \{0, \ldots, N\}^d \right\}. \tag{16}$$

Thus, the grid size is $h_N = 2R/N$ and the grid contains $(N+1)^d$ nodes.

For $u \in K$, let

$$\pi_N(u) := \operatorname*{arg\,min}_{v \in \mathcal{G}_N} |u - v|, \qquad \widetilde{\mu}_0^N := \mu_0 \circ \pi_N^{-1}. \tag{17}$$

Let $h = n^{-1}$, $t_k = kh$, and

$$\kappa_n(s) := t_k, \qquad s \in [t_k, t_{k+1}). \tag{18}$$

The deterministic-grid Euler–Maruyama approximation is

$$\begin{aligned} x_n^N(u,t) = u &+ \int_0^t a(x_n^N(u, \kappa_n(s)), \mu_{\kappa_n(s)}^{N,n})\, ds \\ &+ \int_0^t \int_{\mathbb{R}^d} b(x_n^N(u, \kappa_n(s)), \mu_{\kappa_n(s)}^{N,n}, p)\, W(dp, ds), \end{aligned} \tag{19}$$

where

$$\mu_t^{N,n} := \widetilde{\mu}_0^N \circ \left(x_n^N(\cdot, t)\right)^{-1}. \tag{20}$$

Equivalently, at the grid times,

$$\begin{aligned} x_{n,k+1}^N(u) = x_{n,k}^N(u) &+ h\, a(x_{n,k}^N(u), \mu_k^{N,n}) \\ &+ \int_{t_k}^{t_{k+1}} \int_{\mathbb{R}^d} b(x_{n,k}^N(u), \mu_k^{N,n}, p)\, W(dp, ds), \end{aligned} \tag{21}$$

with

$$x_{n,0}^N(u) = u, \qquad \mu_k^{N,n} = \widetilde{\mu}_0^N \circ (x_{n,k}^N)^{-1}. \tag{22}$$

Theorem 2 *Let* (15) *and* (3)–(4) *hold. Fix $p > d$,*

$$0 < \eta < \frac{p-d}{p}, \qquad \sigma := \frac{\eta}{\eta + d}. \tag{23}$$

Then there exists $C < \infty$, independent of N and n, such that

$$\begin{aligned} &\mathbb{E} \sup_{u \in K} \sup_{t \in [0,1]} |x(u,t) - x_n^N(\pi_N(u), t)| \\ &\quad \le C \left((\gamma_2^2(\mu_0, \widetilde{\mu}_0^N))^\sigma + N^{-\eta} + n^{-1/2} \right). \end{aligned} \tag{24}$$

then

$$\mathbb{E}\sup_{u\in K}\sup_{t\in[0,1]}|x(u,t)-x_n^N(\pi_N(u),t)|\le C\left(N^{-2\sigma}+N^{-\eta}+n^{-1/2}\right). \tag{25}$$

2. Monte Carlo approximation. To reduce the number of spatial nodes required by the discretization procedure and to avoid the dimensional dependence of regular grid constructions, we propose to approximate the initial measure by an empirical Monte Carlo measure

The deterministic measure (17) is replaced by

$$\mu_0^N := \frac{1}{N}\sum_{i=1}^N \delta_{U_i}, \qquad U_1,\dots,U_N \text{ i.i.d. with law } \mu_0, \qquad (U_1,\dots,U_N)\perp W. \tag{26}$$

The Euler–Maruyama–Monte Carlo approximation is

$$\begin{aligned} x_n^N(u,t) = u &+ \int_0^t a(x_n^N(u,\kappa_n(s)),\mu^{N,n}_{\kappa_n(s)})\,ds \\ &+ \int_0^t\int_{\mathbb{R}^d} b(x_n^N(u,\kappa_n(s)),\mu^{N,n}_{\kappa_n(s)},p)\,W(dp,ds), \end{aligned} \tag{27}$$

where

$$\mu_t^{N,n} := \frac{1}{N}\sum_{i=1}^N \delta_{x_n^N(U_i,t)}. \tag{28}$$

At $t_k = kh$,

$$\begin{aligned} x^N_{n,k+1}(u) = x^N_{n,k}(u) &+ h\,a(x^N_{n,k}(u),\mu_k^{N,n}) \\ &+ \int_{t_k}^{t_{k+1}}\int_{\mathbb{R}^d} b(x^N_{n,k}(u),\mu_k^{N,n},p)\,W(dp,ds), \end{aligned} \tag{29}$$

with

$$x^N_{n,0}(u) = u, \qquad \mu_k^{N,n} = \frac{1}{N}\sum_{i=1}^N \delta_{x^N_{n,k}(U_i)}. \tag{30}$$

Assume

$$\mu_0\in M_q(\mathbb{R}^d), \qquad q>2, \tag{31}$$

and

$$q\ne 4 \quad (d\le 4), \qquad q\ne \frac{2d}{d-2} \quad (d>4). \tag{32}$$

Define

$$R_{d,q}(N) := \begin{cases} N^{-1/2}+N^{-(q-2)/q}, & d<4,\\ N^{-1/2}\log(1+N)+N^{-(q-2)/q}, & d=4,\\ N^{-2/d}+N^{-(q-2)/q}, & d>4. \end{cases} \tag{33}$$

Theorem 3 *Under* (31)–(32),

$$\mathbb{E}\,\gamma_2^2(\mu_0,\mu_0^N)\le C(d,q,M_q(\mu_0))R_{d,q}(N). \tag{34}$$

For $\theta\in(0,1)$, set

$$\alpha := \frac{\theta}{2\theta+d}. \tag{35}$$

The two components of the total error satisfy

$$\mathbb{E}\sup_{u\in K}\sup_{t\in[0,1]}|x(u,t)-x^N(u,t)|\le C_K R_{d,q}(N)^\alpha, \tag{36}$$

$$\mathbb{E}\sup_{u\in K}\sup_{t\in[0,1]}|x^N(u,t)-x_n^N(u,t)|\le C_K n^{-\alpha}. \tag{37}$$

Theorem 4 *Let* (3)–(4) *and* (31)–(32) *hold. Let $K\subset\mathbb{R}^d$ be compact and let α be given by* (35). *Then*

$$\begin{aligned}&\mathbb{E}\sup_{u\in K}\sup_{t\in[0,1]}|x(u,t)-x_n^N(u,t)|\\&\quad\le \mathbb{E}\sup_{u,t}|x(u,t)-x^N(u,t)|+\mathbb{E}\sup_{u,t}|x^N(u,t)-x_n^N(u,t)|\\&\quad\le C_K\left(R_{d,q}(N)^\alpha+n^{-\alpha}\right).\end{aligned} \tag{38}$$

The passage from the first result to the second is expressed by

$$\widetilde{\mu}_0^N=\mu_0\circ\pi_N^{-1}\quad\longrightarrow\quad\mu_0^N=\frac{1}{N}\sum_{i=1}^N\delta_{U_i}, \tag{39}$$

$$\operatorname{supp}\mu_0\subset[-R,R]^d\quad\longrightarrow\quad\mu_0\in M_q(\mathbb{R}^d),\qquad q>2, \tag{40}$$

while the total error changes from

$$C\left((\gamma_2^2(\mu_0,\widetilde{\mu}_0^N))^\sigma+N^{-\eta}+n^{-1/2}\right) \tag{41}$$

to

$$C_K\left(R_{d,q}(N)^\alpha+n^{-\alpha}\right). \tag{42}$$

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Pathwise stochastic integration and invariance with respect to the choice of the partition sequence

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In this talk, we present a pathwise approach to stochastic integration: Using the concepts of quadratic variation and Lévy area of a continuous path along a sequence of time partitions, we construct a pathwise integral as a limit of general Riemann sums. In a probabilistic framework, when the underlying process is a semimartingale, this notion of integration is consistent with stochastic integration. Furthermore, we give necessary and sufficient conditions for the quadratic variation and Lévy area of a continuous path to be invariant with respect to the choice of the partition sequence.

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A New Coupling Construction for Markov Chains in Random Environments

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Let E and F be complete separable metric spaces equipped with their Borel σ-fields $\mathcal{B}(E)$ and $\mathcal{B}(F)$, respectively. Let $(X_n)_{n\in\mathbb{Z}}$ be a strictly stationary F-valued process. We consider the recursively defined process

$$Y_{n+1} = f(Y_n, X_n, \varepsilon_{n+1}), \qquad n \in \mathbb{N},$$

where Y_0 is a possibly random initial state, $f : E \times F \times \mathbb{R}^d \to E$ is measurable, and $(\varepsilon_n)_{n\geq 1}$ is a $[0,1]$-valued noise sequence. We assume that $(\varepsilon_n)_{n\geq 1}$ is i.i.d., independent of $(X_n)_{n\in\mathbb{Z}}$, and that Y_0 is independent of $(\varepsilon_n)_{n\geq 1}$. Then $(Y_n)_{n\in\mathbb{N}}$ forms a Markov chain in a random environment (MCRE): conditionally on the exogenous process $(X_n)_{n\in\mathbb{Z}}$, the process $(Y_n)_{n\in\mathbb{N}}$ is a time-inhomogeneous Markov chain whose transition probabilities are modulated by the random environment.

Markov chains in random environments have been extensively studied under Foster–Lyapunov-type drift and minorization conditions, adapted from the theory of general state-space Markov chains (see, e.g., [1], [3] and references therein). These assumptions ensure the existence of suitable small sets in the state space that are visited sufficiently often by the process, and on which coupling occurs with positive probability at each visit.

In this talk, we present an alternative coupling approach, developed in detail in [2]. Instead of relying on the existence of small sets, we assume that the dynamics is contractive at large distances. Moreover, we replace the classical minorization condition by a weaker local version, which guarantees that trajectories of $(Y_n)_{n\in\mathbb{N}}$ that come sufficiently close to each other couple with positive probability.

Under a few additional mild technical assumptions, we establish the existence of a stationary solution $\left((X_n, Y_n^*)\right)_{n\in\mathbb{Z}}$. Furthermore, we prove that the law of (X_n, Y_n) converges in total variation distance to the stationary law $\mathrm{Law}((X_0, Y_0^*))$, and we provide explicit convergence rate estimates. Finally, we derive explicit upper bounds on the α-mixing coefficients of the joint process $(X_n, Y_n)_{n\in\mathbb{N}}$ in terms of the α-mixing coefficients of the environment.

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Superconcentration for minimal surfaces in random environment

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We study the variance of the energy of minimal surfaces in random environment in dimensions up to 4. We consider two different models, introduced by Dembin, Elboim, Hadas, and Peled [3] and by Agresti, Clozeau, and Fischer [1].

We prove an upper bound on the variance of the form

$$\mathrm{Var}[\min E] \lesssim \frac{L^{d-1}}{(\log L)^{\theta}} \tag{1}$$

for some $\theta > 0$, which is a logarithmic improvement over the classical theory (such as spectral gap inequality). Our proof is inspired by the proof of sublinear variance in first-passage percolation by Benjamini, Kalai, and Schramm [2], which we adapt to higher dimensions. This adaptation relies on a quantiative homogenisation result that was obtained by Agresti et. al [1].

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A weak transport approach to the Schrödinger–Bass problem

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The Schrödinger–Bass problem is a semimartingale optimal transport problem connecting the celebrated Schrödinger bridge with the Bass martingale problem. In this talk, I will explain how the dynamic Schrödinger–Bass problem admits an equivalent static formulation as a weak optimal transport problem with an explicit cost. This reformulation yields existence and duality results, provides a structural characterization of the optimal semimartingales, and naturally leads to a Sinkhorn-type iteration for computation. This is joint work with Manuel Hasenbichler and Stefan Thonhauser.

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Learning Distributed Equilibria in Linear-Quadratic Stochastic Differential Games: An α-Potential Approach

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Multi-agent reinforcement learning has shown impressive practical success in solving dynamic games, with applications ranging from autonomous driving and robotics to neuroscience and economics. Despite this empirical progress, theoretical understanding of the convergence of these learning algorithms in dynamic and stochastic environments remains limited.

In this talk, we take a first step toward bridging the gap by studying a fundamental class of N-player dynamic games: linear-quadratic stochastic differential games. We focus on independent policy-gradient learning, where each player employs a distributed policy that depends only on its own state and updates its policy independently by following the gradient of its own objective. We show that the game admits an α-potential structure, where α is determined by the degree of pairwise interaction asymmetry. We further characterize an affine distributed equilibrium beyond the mean field setting for both symmetric and asymmetric interactions. For pairwise-symmetric interactions, we show that independent policy-gradient methods converge globally to an affine equilibrium at a linear rate. For asymmetric interactions, we prove that independent projected policy-gradient algorithms converge linearly to an approximate equilibrium, with the suboptimality proportional to the degree of asymmetry.

Talagrand-type transport inequalities for path spaces over Carnot groups

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Talagrand's transportation inequality relates the Wasserstein distance and relative entropy, and is known to hold, for example, when the reference measure is a standard Gaussian or the law of a Brownian motion on Euclidean space.

We consider Talagrand-type transportation inequalities for the law of Brownian motion on Carnot groups. An important example is the lift of standard Brownian motion to the Brownian rough path. We present a direct proof on enhanced path space, which also yields equality when restricting to adapted couplings in the transport problem. Moreover, we prove a Talagrand inequality for the heat kernel measure on Carnot groups and deduce the inequality for the law of Brownian motion on Carnot groups via a bottom-up argument.

With a non-commutative sub-Riemannian state space, we observe phenomena that differ from the Euclidean case. In particular, while a top-down projection argument recovers Talagrand's inequality on Euclidean space from the corresponding inequality on the path space, such a projection argument breaks down in the Carnot group setting.

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Backward stochastic porous media equation with polynomial growth

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We investigate the well-posedness of a backward stochastic porous media equation whose nonlinear term is a polynomial operator corresponding to the slow diffusion regime. The main challenge stems from the structure of this nonlinear operator, which prevents the use of standard weak–strong convergence techniques to identify the limit.

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The Heisenberg XXZ chain, the six-vertex model and the Lorentz mirror model with loop weight 2

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The Lorentz mirror model is a model of random loops on $\mathbb{Z}^2$: each vertex is independently assigned either has a north-west oriented mirror, a north-east oriented mirror, or no mirror. Loops bounce off mirrors if they are present and pass straight through vertices with no mirror. A longstanding open problem is to prove that there are no infinite loops almost surely.

When one re-weighs this measure by a factor $2^{\#loops}$, the same behaviour is expected, but one also has a very simple coupling to the six-vertex model. Using the recently proved delocalisation of the height function of the six-vertex model, as well as a new coupling, we prove that the probability that two points are connected by a loop decays polynomially fast.

We will also see how this can be used to prove two statements in the 1D quantum Heisenberg XXZ model: that finite-volume, finite-temperature states converge to an infinite volume ground state, and that spin-spin correlations in this ground state decay polynomially fast.

Long-time behaviour for discretization schemes of the Fokker-Planck equation via couplings

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In this talk we construct continuous-time Markov chains associated to finite-volume discretization schemes of Fokker-Planck equations and establish sufficient conditions under which we show quantitative exponential decay in the ϕ-entropy and Wasserstein distance. The decay in ϕ-entropy implies modified logarithmic Sobolev, Poincaré, and discrete Beckner inequalities. The results are not restricted to additive potentials and do not rely on discrete Bochner-type identities. The proof for the ϕ-decay relies on a coupling technique due to Conforti [1], while the proof for the Wasserstein distance uses the path coupling method.

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A Rough Path Approach to Large Matrix Models

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We introduce a rough path approach to proving both, weak and strong convergence of solutions to matrix SDEs and RDEs to their free-probabilistic limits.

This is achieved by developing a version of rough path theory on varying Banach algebras and introducing new notions of weak and strong convergence for non-commutative rough paths. We prove continuity of the associated Itô–Lyons type map: convergence of noncommutative rough paths implies convergence of solutions to the corresponding rough differential equations. We then establish weak and strong convergence of rough paths constructed from GOE(N) and GUE(N) processes.

As part of the work, we for the first time construct canonical rough path lifts of general free semicircular processes in their natural C^*-topology, a long-standing problem in rough path theory.

This is joint work with A. Chandra, I. Chevyrev and M. Peev.

A dice game, a multinomial walk, and the inverted Dirichlet distribution

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We consider a simple dice game on a board, which leads to an intriguing study of multinomial walks, with surprising and seemingly paradoxical properties. The winning and losing probabilities of a general version of the game are investigated via conjugacy relations between Gamma and Poisson distributions, as well as between negative multinomial and inverted Dirichlet distributions. We show a monotonicity property of the regularized incomplete beta function, which implies a monotonicity property of the winning probability. Furthermore, the asymptotic behavior of the game for one or several parameters of the game tending to infinity is analyzed, as well as the probability of being last in the game.

Chain rule and singular SPDEs

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Singular stochastic PDEs are nonlinear equations with a rough stochastic forcing, such that the expected regularity of a solution is not sufficient for the nonlinearity to be well defined. Nevertheless, solutions can be given a sense after a renormalization procedure. In this talk, we want to understand to what extent this renormalization can be chosen such that the solution transforms according to the chain rule – cf. Itô vs. Stratonovich integration. This has been answered in the literature using heavy algebraic tools, whereas our argument is elementary and purely analytic.

Asymptotic Theory for Two-Sample Instrumental Variables Estimators

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Two-sample instrumental variable (IV) methods are widely used in Mendelian randomisation, where researchers estimate causal effects using summary statistics from independent samples. While the inverse variance weighted (IVW) estimator is the standard approach, existing asymptotic theory largely focuses on settings with a growing number of instruments, and its validity can require strong conditions in the presence of many weak instruments.

We develop asymptotic theory for two-sample IV estimators by establishing consistency and asymptotic normality of both IVW and debiased IVW (dIVW) when the number of instruments is fixed, complementing existing many-instrument theory. We show that IVW and dIVW have the same first-order asymptotic variance in this setting. We then compare IVW and dIVW with the profile score (PS) estimator across fixed- and many-instrument regimes. Under standard variance assumptions, dIVW and PS require the same rate of collective instrument strength but impose different additional restrictions on instrument growth and the concentration of instrument strength, so their theoretical guarantees favour different asymptotic regimes. Under a common variance-ratio condition, we further show that dIVW has strictly smaller asymptotic variance than PS in the fixed-instrument setting, while their relative variance difference vanishes as the sample size increases.

These results provide a unified theoretical comparison of the commonly used two-sample IV estimators and clarify how instrument strength, instrument growth, and the concentration of instrument strength determine their validity and relative efficiency.

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Limit Shape of Single-Source Stochastic Sandpiles with p–topplings on $\mathbb{Z}$

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The Abelian sandpile model is a model of mass distribution in which particles accumulate on the vertices of a graph and unstable vertices topple deterministically by distributing particles to their neighbors. We study a stochastic variant of the single-source model on $\mathbb{Z}$, where $n \in \mathbb{N}$ particles are placed at the origin. Unstable vertices topple according to the p–toppling rule, sending a particle to each neighbor independently with probability $p \in (0, 1)$. We show that as $n \to \infty$, the macroscopic limit shape of the final stable configuration is a symmetric interval around the origin and establish a central limit theorem for the boundary fluctuations: after proper rescaling, they converge to a Gaussian distribution.

Inverting Markovian Projections: Weak Error Rates for Local Stochastic Volatility Models

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Local stochastic volatility refers to a popular model class in applied mathematical finance that allows for "calibration-on-the-fly". These models aim to replicate the one-dimensional marginal distributions of local volatility models (which are Markovian SDEs), while still allowing for a stochastic volatility component, which is typically non-Markovian.

The governing SDEs from these models arise from inverting the Markovian projection, originally established by Győsngyi, resulting in coefficients that depend on conditional expectations of the solution itself. While well-posedness of the equation remains a largely open problem, our take is to start with a well-defined Euler approximation.

We then establish a novel half-step-scheme that allows for good approximations of conditional expectations. After introducing a regularisation parameter, we formulate an interacting particle system, transforming it to a well-posed McKean-Vlasov Half-Step Euler scheme. Assuming a solution to the original problem exists, by a mimicking argument we then quantify the weak error rate of the particle system with respect to all parameters involved.

On minimizing curves in a Brownian potential

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In this talk, we consider a semi-discrete random variational problem that can be interpreted as the geometrically linearized version of the critical 2-dimensional random field Ising model. To leading order the minimal energy shows logarithmic corrections to the natural volume scaling. We discuss a quenched homogenisation result in the sense that when properly rescaled this leading order contribution becomes deterministic as the ratio system size / lattice spacing diverges. On the other hand, we establish bounds that yield tightness of the law of minimizers in the space of continuous functions as the lattice spacing is sent to zero.

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Optimal anticoncentration of spanning trees in pseudorandom graphs

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Let G be an n-vertex d-regular graph. Lee established the following anticoncentration inequality for spanning trees: Let $\mathbf{T}$ be an uniformly random spanning tree in T and T be an arbitrary spanning tree on n vertices. Then, $\mathbb{P}(T \simeq \mathbf{T}) \leq \exp(-n/2000)$. In particular, it implies there are $\exp(n/2000)$ non-isomorphic spanning trees in G.

Our main result gives the asymptotically optimal anticoncentration bound under additional spectral constraints: For every $\varepsilon > 0$, there exists $C > 0$ such that if G is an (n, d, λ)-graph with $d/\lambda \geq C$, then

$$\mathbb{P}(T \simeq \mathbf{T}) \leq \exp(-(1-\varepsilon)n)\,.$$

This resolves an open problem from Lee's paper and the bound is asymptotically optimal.

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Random subgraphs

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Given a graph G and a real number $p \in [0,1]$, we can form a random subgraph G_p by keeping each edge of G independently with probability p. This general definition captures many classical notions, including the binomial random graph $G_{n,p}$ (which corresponds to the case $G = K_n$) and Bernoulli bond percolation (where G is an infinite transitive graph, such as the lattice $\mathbb{Z}^d$).

In this talk, I will present some new results on this random object, in which we assume essentially nothing about the graph G apart from its average degree d. In particular, I will discuss what happens around the critical probability $p \asymp 1/d$, discussing the emergence of complex structures such as large connected components and long cycles.

Based on joint works with subsets of Micha Christoph, Patryk Morawski, and Alp Müyesser.

Beyond the positive drift: Comparing historical and current daily temperature patterns based on two sample statistics for unbalanced dense-sparse functional data

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The two-sample problem for functional data is investigated for discrete, synchronous designs in each sample, in settings in which one sample is densely observed while the other is only relatively sparsely observed. This is motivated by comparing historical and more current daily temperature patterns, where more recent devices take measurements every 10 minutes, while historical measurements in the time period 1952 to 1972 are available only every hour. We use recently developed methods from transfer learning for functional data to estimate the difference of the mean functions at optimal rates in the supremum norm. Further, we derive a central limit theorem in the space of continuous functions and discuss the construction of uniform confidence bands using the multiplier bootstrap. We also show how our methods can be extended to functional time series. In the application to daily temperature patterns we decompose the mean difference function into a daily average - the normalized integral of the mean difference function - as well as into the deviation from the average value. Using the developed inferential methodology we show that not only the daily average temperatures in each month have increased significantly, but also that the daily temperature patterns have changed for most months, with night temperature remaining relatively stable while daily temperatures increasing beyond the daily average value.

An Occupation-Time Approach to Relative Drawdown

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We study a stochastic control problem motivated by portfolio management under relative performance evaluation. An investor's wealth process is compared against a benchmark - an exogenous, non-replicable reference process (e.g., a stock market index) - and the quantity of interest is the benchmark-relative log-drawdown, the shortfall of the (log) wealth-to-benchmark ratio below its running maximum. Rather than penalizing the magnitude of this drawdown, we penalize its *persistence*: the objective is the expected discounted occupation time that the drawdown process spends beyond a fixed critical threshold.

Although the drawdown introduces path dependence through its running-maximum term, we show that the problem reduces to a one-dimensional Markovian control problem and derive the corresponding Hamilton-Jacobi–Bellman equation. The optimal feedback control admits an explicit characterization as a metric projection of a state-dependent vector onto a convex constraint set. We prove a verification theorem and give geometric conditions under which the resulting reflected diffusion possesses a unique strong solution.

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