Book of Abstracts

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Plenary talks

Reforming Public Pensions: Probabilistic perspectives

Julia Eisenberg

TU Wien

Pay-as-you-go (PAYG) pension systems are particularly vulnerable to demographic risks. This raises serious concerns about the long-term sustainability of public pension systems that rely on a PAYG financing model, where current contributions fund current pension expenditures.

In this talk, we explore two potential approaches to reforming such systems.

The first approach assumes that any deficit in the PAYG scheme is immediately covered by the state. In return, individuals are required to invest a specified amount into a fund. This investment is structured such that individuals are expected, with a certain probability, to repay the state for the deficit incurred by the PAYG scheme. We compare the outcomes of this approach with those of the traditional direct contribution model and identify the conditions under which the proposed scheme would be advantageous.

The **second scenario** introduces a mixed pension framework (PAYG and funding) tailored for economies with a declining working-age population, operating under a defined benefit assumption. In this setup, we analyse the consequences of guaranteeing a zero return on the investments made in the funded component. Although this mixed system does not provide a hedge against demographic risks, it serves to delay the need for immediate pension reforms.

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Abelian sandpiles and random walks

 $Ecaterina\ Sava ext{-}Huss$

University of Innsbruck

The abelian sandpile has been introduced in physics by Bak-Tang-Wiesenfeld in 1987 as a model based on simple rules of mass distribution which exhibits peculiar properties and beautiful dynamics, and which is from the mathematical point of view hardly understood. I will introduce the model and state some of its main properties, and show how to understand it as a random walk on a finite group. As such, once can ask about stationary distributions and mixing times, and these are some of the questions that I will address during the talk. Moreover, I will also speak about stabilization/explosion of sandpiles on state spaces of fractal nature and I will state several open questions. The talk is based on joint recent works with Robin Kaiser and Nico Heizmann.

Contributed talks

From Models and Methods – A Practitioner's View on Financial Mathematics

Michael Aichinger

uni software plus GmbH

Financial markets demand models that are both theoretically sound and operationally viable. This talk offers a practitioner's perspective on the evolution and application of risk factor modeling-starting fromsimple flat volatility assumptions and progressing toward sophisticated jump-diffusion and pure jump models. We explore how these models are used across asset classes such as equities, FX, commodities, and electricity, where phenomena like fat tails, pricejumps, and regime shifts require careful treatment. Drawing on real-world experience, we examine the challenges of model calibration-highlighting issues such as parameter instability and overfitting. On the methodological front, we discuss numerical techniquesincluding Monte Carlo and Quasi Monte Carlo simulation, as well as methods based on partial (integro-)differential equations. Each comes with trade-offs in terms of performance, stability, and suitability for specific problem classes. Finally we address applications valuation, scenario analysis and risk management.

Optimal Dividends for an Ornstein-Uhlenbeck surplus

Fabio Colpo and Julia Eisenberg

TU Wien

We consider an insurance company whose surplus follows an Ornstein-Uhlenbeck (OU) process driven by a standard Brownian motion. The company pays dividends to its shareholders and seeks to maximise the expected value of the future discounted dividends. Late dividend payments are penalised/rewarded not only through the usual discounting, but through an additional exponential factor.

We find the optimal strategy for the case of mean-reverting and non-mean-reverting OU processes and illustrate our findings by a numerical example.

- [1] S. Asmussen and M. Taksar: Controlled diffusion models for optimal dividend pay-out, *Insurance: Mathematics and Economics*, (1997), 20(1):1–15.
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- [4] J. Eisenberg: Unrestricted consumption under a deterministic wealth and an Ornstein-Uhlenbeck process as a discount rate, *Stochastic models*, (2018), 34(2):139–153.
- [5] F. Locas and J.-F. Renaud: De Finetti's control problem with a concave bound on the control rate, Journal of Applied Probability, (2024), pages 1–17.
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One Theorem, 16 Couplings

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 - ^b Tsinghua University
- ^c Princeton University

Every first year probability student knows that one may shed light on a joint probability law by computing its marginals, but that this does not determine the joint law uniquely. Conversely, one may gain insight about a given probability law by understanding the joint laws that have it as a marginal. Such arguments are generally referred to as coupling methods, and they are of vital importance in the study of various statistical mechanics models such as random walks and the Ising model. As such, it is of inherent interest to simply understand criteria under which couplings exist. In this talk, we will discuss classical couplings between the Ising model and its graphical representations as well as try to give a flavour of the arguments that they help one accomplish. Finally, we will discuss a theorem which has all the classical couplings of the Ising model as special cases, and which yields at least three novel couplings.

References:

[1] U. T. Hansen, J. Jiang and F. R. Klausen: A General Coupling For Ising Models and Beyond, arXiv preprint arXiv:2506.10765, 2025.

Classification of extremal stationary measures for multi-class ASEP and stochastic six vertex model

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The asymmetric simple exclusion process and stochastic six vertex model are classical interacting particle systems. Their multi-class generalizations are natural generalizations stemming from a shared property called attractivity.

We show that all extremal stationary measures of the multi-class stochastic six vertex model (with shift) or of ASEP are given either by a projections of the ASEP speed process or the q-Mallows measure.

Almost sure convergence rates of adaptive increasingly rare Markov chain Monte Carlo

Julian Hofstadler

University of Bath

We consider adaptive increasingly rare Markov chain Monte Carlo (AIR MCMC), which is an adaptive MCMC method, where the adaptation concerning the "past" happens less and less frequently over time. We are interested in the convergence behaviour of renormalised Monte Carlo sums and show limit results which hold under a Wasserstein contraction assumption. Our results hold in an almost sure setting and we obtain rates which are close to those in the law of the iterated logarithm.

This talk is based on joint work with K. Łatuszyński, G. Roberts and D. Rudolf.

Maximal Displacement of Supercritical Branching Random Walks on Free Products of Finite Groups

Robin Kaiser

University of Innsbruck

In this talk, we will discuss the question of how far the particles of a supercritical branching random walk travel from their starting position, which is a quantity known as the maximal displacement of the branching random walk. This question has been extensively studied on the real line, where it is known that the maximal distance to the starting vertex grows linearly in time, and even the second order correction term has been determined to be of logarithmic order. However, on other state spaces, our understanding of the maximal distance travelled by the particles of a supercritical branching random walk is far less clear than on the real line.

In my presentation, I will discuss the maximal displacement of supercritical branching random walks on free products of finite groups, where we are able to show that the maximal distance travelled also grows linearly in time almost surely. We will see how the maximal displacement relates to large deviation estimates of the underlying step distribution, and how we can use such estimates to prove the linear growth of the maximal displacement.

Exact simulation of the first-passage time of SDEs to time-dependent thresholds

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 c Institute of Stochastics, Johannes Kepler University Linz

The first-passage time (FPT) is a fundamental concept in stochastic processes, representing the time it takes for a process to reach a specified threshold for the first time. Often, considering a time-dependent threshold is essential for accurately modeling stochastic processes, as it provides a more accurate and adaptable framework. In this work, we extend an existing Exact simulation method developed for constant thresholds to handle time-dependent thresholds. Our proposed approach utilizes the FPT of Brownian motion and accepts it for the FPT of a given process with some probability, which is determined using Girsanov's transformation. This method eliminates the need to simulate entire paths over specific time intervals, avoids time-discretization errors, and directly simulates the first-passage time. We present results demonstrating the method's effectiveness, including the extension to time-dependent thresholds, comparisons with existing methods through numerical examples.

References:

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- [2] S. Herrmann and C. Zucca: Exact simulation of the first-passage time of diffusions., Journal of Scientific Computing, 79:1477–1504, 2019.

Asymptotic equivalence of non-parametric regression on spherical *t*-designs and Gaussian white noise

M. Kroll

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The notion of spherical t-designs, introduced in the seminal paper [1], has attracted interest in various areas of mathematics over the past decades. In this work, we consider spherical t-designs as the set of sampling points in a fixed design non-parametric regression on spheres of arbitrary dimension. We show that the fixed design regression experiments defined this way are asymptotically equivalent, in the sense of Le Cam, to a sequence of Gaussian white noise experiments as the sample size tends to infinity. More precisely, asymptotic equivalence is established for Sobolev and Besov function classes on the sphere. These results provide further support for the use of spherical t-designs as sampling points in non-parametric regression with spherical regressors.

References:

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Stochastic Simulations of Chemical Reaction Systems Using Rule-Based Graph Models

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Modeling chemical reaction systems using sets of coupled ordinary differential equations (ODEs), which are then solved numerically, can be effective for small-scale reaction networks. However, as systems become larger, more complex, or exhibit combinatorial behavior, this deterministic approach often fails to capture important features such as concurrent reaction events, strong nonlinearities, or possible alternative reaction pathways—common characteristics of such systems. Stochastic simulation methods, like Gillespie-type algorithms [1], offer a more suitable alternative by accounting for randomness in reaction dynamics. Yet, these methods still require prior knowledge of the complete reaction network. Rule-based modeling [2] addresses this limitation by enabling network-free or on-the-fly stochastic simulations [3]. Instead of relying on a predefined reaction network, this approach operates on a set of reaction rules, each with associated rates. As the simulation progresses, the reaction network is generated dynamically based on the application of these rules. In the case of chemical reactive systems, molecules are modeled as graphs, and reactions are implemented as graph transformation rules [4]. This framework allows for rate calculations that can depend on the structural properties of molecules—enabled through customizable callback functions. Additionally, these rate estimates can be further refined using onthe-fly calculations of physicochemical properties, such as energies of formation. This modeling approach was applied to study the recently introduced glyoxylose chemical space [5], a system structurally analogous to the prebiotically significant formose reaction. The results of this investigation will be presented in this talk. Overall, this foundational modeling framework holds promise for future studies aimed at estimating kinetic parameters in experimental chemical systems.

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^b Algorithmic Cheminformatics Group, University of Bielefeld, Germany

When risk defies order: The nonexistence of monetary risk measures under fractional stochastic dominance

Christian Laudagé a , and Felix-Benedikt Liebrich b

In this talk, we study the existence and structure of monetary risk measures consistent with fractional stochastic orders, a refinement of second-order stochastic dominance defined via a threshold utility function v, whose absolute risk aversion determines the set of test utilities. Motivated by applications in different risk sharing setups, we derive representation results for this class. However, the deeper issue is whether such risk measures exist at all. Our results indicate that they are either highly constrained or non-existent, except when v is given as exponential utility function. Imposing further properties such as convexity or positive homogeneity makes existence even less likely. In addition, we outline implications for practical risk management.

Stochastic parabolic equations with singular potentials

T. Levajković

Institute of Statistics and Mathematical Methods in Economics, TU Vienna

This talk addresses a broad class of stochastic parabolic partial differential equations (PDEs) involving singular potentials, where the potential terms can be highly irregular. We consider several types of such equations, motivated by a range of applications from finance, structural mechanics, fluid dynamics and biology, where modeling uncertainty through stochastic effects is essential. To rigorously study these problems, we employ the Wick product from white noise analysis, a regularization technique that enables meaningful multiplication of generalized stochastic processes. The analysis is carried out within the framework of chaos expansions and combines tools from white noise analysis with the concept of very weak solutions in the theory of PDEs. For each class of equations considered, we formulate a notion of a stochastic very weak solution and establish results on existence and uniqueness. Moreover, we demonstrate that when the potential and input data are sufficiently regular, the stochastic very weak solution agrees with the classical stochastic weak solution. The talk is based on the joint work with Ljubica Oparnica and Snežana Gordić.

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Transition of α -mixing in Random Iterations

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Nonlinear time series models with exogenous regressors play a pivotal role in econometrics, queuing theory, and machine learning, offering robust frameworks for modeling and prediction. Despite their widespread utility, the statistical analysis of such models remains challenging, particularly when dealing with non-i.i.d. exogenous covariate processes.

Fundamental results, including the law of large numbers, the central limit theorem, and concentration inequalities, have been established for weakly dependent variables. We demonstrate the transfer of mixing properties from exogenous regressors to the response variable through coupling arguments, making applicable these results, thus providing a powerful tool for analyzing nonlinear autoregressions with exogenous inputs.

Furthermore, we investigate Markov chains in random environments under suitable versions of drift and minorization conditions, including non-stationary environments with favorable mixing properties. This framework is then applied to single-server queuing models, opening door to the statistical analysis of waiting times.

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- [2] A. Lovas and M. Rásonyi: Ergodic theorems for queuing systems with dependent inter-arrival times, *Operations Research Letters*, 49 (2021), 682–687.
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American option pricing using generalised stochastic hybrid systems

Evelyn Buckwar, Sascha Desmettre, Agnes Mallinger, and Amira Meddah

Johannes Kepler University Linz

In this talk, we present a novel approach to pricing American options using piecewise diffusion Markov processes (PDifMPs), a generalised stochastic hybrid system combining continuous dynamics with discrete jumps. Standard models often assume constant drift and volatility, limiting their ability to capture the erratic nature of financial markets. Our method leverages PDifMPs to incorporate sudden market fluctuations, offering a more realistic asset price model. We benchmark our approach against the Longstaff-Schwartz algorithm, including its modified version with PDifMP-based asset price trajectories. Numerical simulations show that the PDifMP method reflects market behaviour more accurately while enhancing computational efficiency. The results suggest that PDifMPs significantly improve American options pricing by aligning with real-world stochastic volatility and market jumps, providing a more predictive and efficient approach.

On the Geometry of Causal Transport

G. Pammer

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Causal transport, together with the associated adapted Wasserstein distance, induces a natural geodesic structure on the space of stochastic processes. In this talk, we present recent advances in the study of the resulting geometry. In particular, we provide a characterization of absolutely continuous curves w.r.t. the adapted Wasserstein distance and further elucidate the geometric structure of Gaussian processes in discrete time.

This talk is based on joint works with B. Acciaio, D. Bartl, M. Beiglböck, S. Hou, D. Krsek, M. Rodriguez, and S. Schrott.

Efficient Construction, Estimation, and Summaries of Ranked Unlabelled Trees using Markov Chains

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We introduce a new framework for the construction, simulation, and statistical analysis of ranked, unlabelled trees—combinatorial structures whose state-space grows over-exponentially (following the Euler zigzag numbers). Building on the F-matrix representation of Salmyak & Palacios (2024) [1], we recast each tree as a sequence of column-vectors that inherit the Markov property, thereby defining a much smaller, tractable state space. Specifically, for a tree with n leaves, the number of distinct column states is Fib(n), and the corresponding transition matrix is a (relatively) sparse Fib(n)×Fib(n) array that admits highly efficient storage and computation.

Within this column-state framework, we derive explicit algorithms for a Markov chain that sequentially builds ranked unlabelled trees. Once assembled, these trees support a variety of inferential procedures. We develop hypothesis tests to assess whether observed transition probabilities align with those of the Kingman coalescent, or come from a more complex model, like the Blum-Francois [2]. Moreover, we discuss tree-appropriate summary statistics — namely, the Fréchet mean and variance on tree space — that succinctly characterize collections of trees. Finally, we present Vitreebi, a dynamic-programming algorithm that computes Fréchet means under our metric, providing a principled summary of sample heterogeneity.

Our approach dramatically reduces computational complexity, opening the door to scalable simulation and inference on large tree-shaped data without sacrificing mathematical rigor or probabilistic fidelity.

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Gumbel fluctuations for Hausdorff approximation by random inscribed polytopes

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Let K be a smooth convex body, that is, a compact convex set with nonempty interior and three times continuously differentiable boundary. Assume that everywhere on the boundary the Gaussian curvature is positive. We can approximate K by the convex hull K_n of n random independent identically distributed points sampled from its boundary ∂K . It is known that the Hausdorff distance between K and the approximation K_n tends to zero as n tends to infinity and the speed of this convergence was determined by Glasauer and Schneider [1]. In case the points are distributed according to the optimal density (which is given in terms of the curvature), we prove that the rescaled Hausdorff distance between K and K_n tends to a Gumbel distributed random variable. The proof is based on an asymptotic relation to the covering properties of random geodesic balls on ∂K and relies on a theorem due to Janson [2].

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Worst-Case Optimal Investment in Incomplete Markets

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We study and solve the worst-case optimal portfolio problem of an investor with logarithmic preferences facing the possibility of a market crash. Our setting takes place in a Lévy-market and we assume stochastic market coefficients. To tackle this problem, we enhance the martingale approach developed in [1]. A utility crash-exposure transformation into a backward stochastic differential equation (BSDE) setting allows us to characterize the optimal indifference strategies. Further, we deal with the question of existence of those indifference strategies for market models with an unbounded market price of risk. To numerically compute the strategies, we solve the corresponding (non-Lipschitz) BSDEs through their associated PDEs and need to analyze continuity and boundedness properties of CIR forward processes. We demonstrate our approach for Heston's stochastic volatility model, Bates' stochastic volatility model including jumps, and Kim-Omberg's model for a stochastic excess return.

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On optimal strong approximation of SDEs with Hölder continuous drift coefficient

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We study strong approximation of the solution of a scalar stochastic differential equation (SDE)

$$dX_t = \mu(X_t) dt + dW_t, \quad t \in [0, 1],$$

$$X_0 = x_0$$
(1)

at the final time point 1 in the case that the drift coefficient μ is α -Hölder continuous with $\alpha \in (0,1]$. Recently, it has been shown in [1] that for such SDEs the equidistant Euler approximation achieves an L^p -error rate of at least $(1 + \alpha)/2$, up to an arbitrary small ε , in terms of the number of evaluations of the driving Brownian motion W. In this talk we present a matching lower error bound. More precisely, we show that the L^p -error rate $(1 + \alpha)/2$ can not be improved in general by no numerical method based on finitely many evaluations of W at fixed time points. For the proof of this result we choose μ to be the Weierstrass function and we employ the coupling of noise technique recently introduced in [2].

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