Have you met the Gaussian free field? (Invited Talk)

Johan Aru
École Polytechnique Fédérale de Lausanne

The Gaussian free field (GFF) can be seen a random height function. It first popped up in the 1960s in the context of Euclidean quantum field theories and started to gain popularity in the probability community from 2000s onwards, due to its various roles in the probabilistic study of statistical physics models. In this overview(ish) talk we will discuss some settings where one can meet this GFF and try to look at some of its properties.

Self-similar Gaussian Markov processes

Benedict Bauer
Universität Wien

We characterize all finite dimensional centered self-similar Gaussian Markov processes via the structure of their covariance kernels. In the one dimensional case this leads to a two parameter family of Gaussian Markov processes such that any centered self-similar Gaussian Markov process is a constant multiple of a process from this family. This yields short and easy proofs of some non-Markovianity results concerning variants of fractional Brownian motion (most of which are known). In the proof, we relate the covariance kernels to certain growth bounded one-parameter matrix semigroups.

Signature methods in stochastic portfolio theory (Invited Talk)

Christa Cuchiero
Universität Wien

In the context of stochastic portfolio theory we introduce a novel class of portfolios which we call linear path-functional portfolios. These are portfolios which are determined by certain transformations of linear functions of a collections of feature maps that are non-anticipative path functionals of an underlying semimartingale. As main example for such feature maps we consider (random) signature of the (ranked) market weights. We prove that these portfolios are universal in the sense that every continuous (possibly path-dependent)

portfolio function of the market weights can be uniformly approximated by signature portfolios. We also show that signature portfolios can approximate the log-optimal portfolio in several classes of non-Markovian models arbitrarily well and illustrate numerically that the trained signature portfolios are remarkably close to the theoretical log-optimal portfolios. This applicability to non-Markovian markets makes these portfolios much more general than classical functionally generated portfolios usually considered in stochastic portfolio theory. Besides these universality features, the main numerical advantage lies in the fact that several optimization tasks like maximizing expected logarithmic utility or mean-variance optimization within the class of linear path-functional portfolios reduces to a convex quadratic optimization problem, thus making it computationally highly tractable. We apply our method to real market data and show generic out-performance on out-of-sample data even under transaction costs. The talk is based on joint work with Janka Möller.

The growth-fragmentation embedded in space-filling SLE_6–explorations of the $\sqrt{8/3}$–quantum disk

William Da Silva
Universität Wien

Branching structures provide a natural framework to understand some features of random geometry. An example of such a connection was revealed in a beautiful work of Bertoin, Budd, Curien and Kortchemski, where a remarkable branching structure appears in the scaling limit from a peeling exploration of some random planar maps. More recently, Miller, Sheffield and Werner constructed the same branching structure directly in the continuum, in the setting of Liouville quantum gravity. The purpose of this talk is to show that similar branching structures can be simply embedded in a planar Brownian motion. We will focus on some recent advances, obtained in a joint work with Ellen Powell and Alexander Watson, towards understanding the $\sqrt{8/3}$–Liouville quantum gravity boundary case. Our arguments will involve only planar Brownian motion and excursion theory.

Two phase transition in the planar Ashkin-Teller Model

Moritz Dober
Universität Wien

The Ashkin-Teller model may be viewed as a pair of interacting Ising models. In the symmetric case, there are two coupling constants J, U where J describes the interaction in both Ising models and U the interaction between the two of them. For $U > J$, the model is conjectured to admit two phase transitions. This has been proved in a perturbative regime about 40 years ago by C.-E. Pfister. We explain how to derive this statement in the whole region $U > J$ by using connections to the six-vertex and further to the random-cluster models via probabilistic couplings.

This talk is based on joint work with Yacine Aoun and Alexander Glazman.

Optimal Reinsurance with Regime-Switching

Julia Eisenberg
TU Wien

We consider a company who wishes to determine the optimal reinsurance strategy minimising the total expected discounted amount of capital injections needed to prevent the ruin. The economy is assumed to evolve in business cycles (modelled by a 2-state Markov chain), impacting the reinsurance price. The constant "once and forever" strategies turn out to be suboptimal. We develop a recursive approach that allows to determine the optimal reinsurance strategies in dependence on the current business cycle.

Statistical consistent energy term structures have affine geometry

Paul Eisenberg
WU Wien

We consider a finite dimensional manifold of possible yield curves for the energy futures market and leave the possibility to estimate the diffusion coefficient from data "freely". Freely means, that the diffusion may take any value which is tangential to the manifold and that there is a corresponding drift coefficient such that no arbitrage is introduced in the market. We show that this implies that the manifold is a submanifold of an affine space of the same dimension. In particular, possible yield curves can always be written as a finite linear combination of base curves.

Analysis of the Ensemble Kalman–Bucy Filter for correlated observation noise

Sebastian Ertel
TU Berlin

The Ensemble Kalman–Bucy filter (EnKBF) is an important tool in the field of stochastic filtering, that aims to approximate the law of a diffusion process, called the signal, conditioned on noisy observations. This is achieved by employing a system of diffusion processes interacting through their ensemble mean and covariance.

In this talk we first derive an EnKBF applicable to the correlated noise framework, that is when the evolution of the signal and the observation process are both influenced by a common noise term.
We prove the well-posedness of the EnKBF, which in the correlated case requires controlling the (pseudo)inverse of the ensemble covariance matrix. Finally we investigate the mean-field limit, which is given by a McKean–Vlasov equation, that only satisfies local Lipschitz condition. We prove the well-posedness of the equation and a propagation of chaos result.

Sanov-type large deviations and conditional limit theorems for high-dimensional Orlicz balls

Lorenz Frühwirth
TU Graz

In this paper, we prove a Sanov-type large deviation principle for the sequence of empirical measures of vectors chosen uniformly at random from an Orlicz ball. From this level-2 large deviation result, in a combination with Gibbs conditioning, entropy maximization and an Orlicz version of the Poincaré-Maxwell-Borel lemma, we deduce a conditional limit theorem for high-dimensional Orlicz balls. Roughly speaking, the latter shows that if V_1 and V_2 are Orlicz functions, then random points in the V_1-Orlicz ball, conditioned on having a small V_2-Orlicz radius, look like an appropriately scaled V_2-Orlicz ball. In fact, we show that the limiting distribution in our Poincaré-Maxwell-Borel lemma, and thus the geometric interpretation, undergoes a phase transition depending on the magnitude of the V_2-Orlicz radius.

The volume of intersections of p-ellipsoids

Michael Juhos
Universität Passau

Motivated by classical works of Schechtman and Schmuckenschläger on intersections of ℓ_p-balls and recent ones in information-based complexity relating random sections of ellipsoids and the quality of random information in approximation problems, we study the threshold behavior of the asymptotic volume of intersections of generalized p-ellipsoids. The non-critical behavior is determined under a spectral flatness (Wiener entropy) condition on the semi-axes. In order to understand the critical case at the threshold, we prove a central limit theorem for q-norms of points sampled uniformly at random from a p-ellipsoid, which is obtained under Noether's condition on the semi-axes.

Quantifying the almost sure uniform convergence of eigenvalue-counting functions

Max Kämper
TU Dortmund

This talk will introduce the concept of almost-additive functions on lattices with the special case of eigenvalue-counting functions of random Schrödinger operators and showcase how they can be used in conjunction with some results from empirical process theory to find explicit error estimates for their convergence to the integrated density of states. This talk is based on joint work with Christoph Schumacher, Fabian Schwarzenberger and Ivan Veselič.

Stability, uniqueness and existence of solutions to McKean-Vlasov SDEs

Alexander Kalinin
Universität München

By extending a method developed by Yamada and Watanabe, we establish Lyapunov stability and pathwise uniqueness of solutions to McKean-Vlasov

equations with random coefficients, which may fail to be locally Lipschitz continuous or of affine growth. Our approach does not rely on the construction of Lyapunov functions and yields explicit Lyapunov exponents for first moment and pathwise exponential stability. Moreover, in the case of deterministic coefficients we combine a sharp first moment estimate with a fixed-point result for time evolution operators to derive unique strong solutions.

Law invariance under heterogeneous beliefs

Felix Liebrich
Leibniz Universität Hannover

Law invariance is one of the key assumptions in the mathematical theory of risk and establishes a close link to probability theory. Given a financial position X modelled as a random variable, law invariance means that the value or risk of X is only driven by the *distribution* of said random variable under a reference probability measure P. Almost all metrics used in practice to measure and manage risk are law invariant. Moreover, a rich strand of literature on the topic has uncovered the far-reaching analytic consequences of this innocent-looking assumption. Very recently, the fact that the reference measure P is treated as given has been problematised, be it for heterogeneous beliefs different financial agents may entertain, the phenomenon of Knightian uncertainty, or scarcity of information that precludes working with a sufficiently determined distribution. In the talk, I will therefore take a concrete functional For wide classes of functionals, I demonstrate that this is not the case *unless* they are (i) constant, or (ii) more generally depend only on the essential supremum and essential infimum of the argument X. In a second part, I will consider the existence of optimal risk sharing procedures among agents entertaining heterogeneous beliefs.

Prediction of random variables by excursion metric projections

Vitalii Makogin
Universität Ulm

In the talk, we use the concept of excursions for the prediction of random variables without any moment existence assumptions. To do so, an excursion metric on the space of random variables is defined, which appears to be a kind of weighted L^1-distance. Using equivalent forms of this metric and the specific choice of excursion levels, we formulate the prediction problem as a minimization of a certain target functional, which involves the excursion metric. Existence of the solution and weak consistency of the predictor are discussed. An application to the extrapolation of stationary heavy-tailed random functions illustrates the use of the aforementioned theory. Numerical experiments with the prediction of Gaussian, α-stable and further heavy–tailed time series round up the talk.

Random walks in view of commensurated subgroups.

Hanna Oppelmayer
Universität Innsbruck

Associated to a random walk μ on a locally compact second countable group G there is the so-called Furstenberg Poisson boundary, which models precisely the bounded μ-harmonic functions on the group. Measure theoretic G-factors of this probability G-space are called μ-boundaries. We provide examples of *prime* measured groups (G, μ), i.e. with no non-trivial μ-boundaries. Furthermore, the quasi-regular representation in these examples turns out to be reducible, disproving a conjecture of Bader-Muchnik. We establish this by using commensurated subgroups and a corresponding homogeneous setting. This provides a general method to pass between certain random walks on dense subgroups of totally disconnected groups to random walks on the ambient group such that their Furstenberg Poisson boundaries are nicely related.
This is joint work with Michael Björklund (Chalmers, Sweden) and Yair Hartman (BGU, Israel).

A Brenier theorem for optimal weak transport

Gudmund Pammer
ETH Zürich

The optimal weak transport problem has recently been introduced by Gozlan et al. While the original motivation stems from the theory of geometric inequalities, the weak transport problem also has been used in different fields such as mathematical finance, economics, and machine learning. The goal of the talk is to familiarize the audience with the foundations of the theory especially in perspective to optimal transport. Finally we discuss as an intriguing application the Brenier-Strassen Theorem by Gozlan-Juillet.

Risk models driven by marked Hawkes processes

Simon Pojer
TU Graz

One way to generalize the classical Cramér-Lundberg model is to replace the Poisson process with a more general Cox process, i.e. a counting process driven by a stochastic intensity. In our model, we consider a risk process, whose counting process N is a marked Hawkes process. This process is characterized by its stochastic intensity, which admits the form $\lambda_t = \nu + \sum_{i=1}^{N_t} h(t - T_i, Y_i)$ for some non-negative function h. Using a cluster representation of this risk process, we can bound it pathwise by a Cramér-Lundberg process. By this, we can show that under the usual assumptions there exists an exponentially decaying upper bound for the corresponding ruin probability. Shifting the initial capital by some constant, we see that the ruin probability of the model can be bounded from below by the ruin probability of the Cramér-Lundberg process multiplied with some positive constant. Consequently, there is also a lower bound, which decays exponentially at the same rate as the upper bound.

Clustering financial institutions based on Wasserstein distance

Lorenz Riess
Universität Wien

Financial institutions submit data on their credit portfolios to regulators. An individual institution can be identified with a distribution that is representative of its respective credit portfolio. We are interested in finding representative clusters of financial institutions based on the notion of Wasserstein barycenter. A particular challenge arises from missing data since financial institutions are subject to different regulatory requirements. This leads us to establish a form of the k-means clustering algorithm in Wasserstein space which can deal with missing coordinates. This is based on joint work with Julio Backhoff and Mathias Beiglböck.

Adapted Wasserstein distances between the laws of SDEs

Ben Robinson
Universität Wien

We study an adapted optimal transport problem between the laws of Markovian stochastic differential equations (SDE) and establish the optimality of the synchronous coupling between these laws. The proof of this result is based on time-discretisation and reveals an interesting connection between the synchronous coupling and the celebrated discrete-time KnotheRosenblatt rearrangement. We also prove a result on equality of topologies restricted to a certain subset of laws of continuous-time processes.

Sandpiles on the gasket

Ecaterina Sava-Huss
Universität Innsbruck

Sandpiles are models of self-organized criticality in which, on a given state space, a configuration of particles is considered. If the number of particles at a site exceeds a given threshold, then the pile is unstable and topples by sending particles to neighboring sites, which in turn may become unstable as well. We proceed until there are no more unstable sites, and we are interested in the limit shape of the set of sites that have toppled in the process of stabilization. We investigate (divisible and abelian) sandpiles on the Sierpinski gasket by giving limit shape results, and bounds on the mixing time for the Abelian sandpile Markov chains.

On the existence of weak solutions to stochastic Volterra equations

David Scheffels
Universität Mannheim

The existence of weak solutions is established for stochastic Volterra equations with timeinhomogeneous coefficients and with convolutional or bounded kernels. The presented approach is based on a newly formulated local martingale problem associated to stochastic Volterra equations.

The Wasserstein space of stochastic processes in continuous time

Stefan Schrott
Universität Wien

Researchers from different areas have independently defined extensions of the usual weak topology between laws of stochastic processes. This includes Aldous' extended weak convergence, Hellwig's information topology and convergence in adapted distribution in the sense of Hoover-Keisler. We show that on the set of continuous processes with canonical filtration these topologies coincide and are metrized by a suitable *adapted Wasserstein distance* $\mathcal{AW}$. Moreover we show that the resulting topology is the weakest topology that guarantees continuity of optimal stopping.

While the set of processes with natural filtration is not complete, we establish that its completion consists precisely in the space processes with filtration FP.

We also observe that $(FP, \mathcal{AW})$ exhibts several desirable properties. Specifically, $(FP, \mathcal{AW})$ is Polish, Martingales form a closed subset and approximation results like Donsker's theorem extend to $\mathcal{AW}$.

This talk is based on joint work with Daniel Bartl, Mathias Beiglböck, Gudmund Pammer and Xin Zhang.

Global contractivity for Langevin dynamics with distribution-dependent forces

Katharina Schuh
Universität Bonn

We study the long-time behaviour of both the classical second-order Langevin dynamics and the nonlinear second-order Langevin dynamics of McKean-Vlasov type. By a coupling approach, we establish global contraction in an L^1-Wasserstein distance with an explicit dimension-free rate for pairwise weak interactions. For external forces corresponding to a κ-strongly convex potential, we obtain a contraction rate of order $\mathcal{O}(\sqrt{\kappa})$ in certain cases. But the contraction result is not restricted to these forces. It rather includes multi-well potentials and non-gradient-type external forces as well as non-gradient- type repulsive and attractive interaction forces. The proof is based on a novel distance function which combines two contraction results for large and small distances and uses a coupling approach adjusted to the distance.

A jump-adapted higher order scheme for jump-diffusion SDEs with discontinuous drift

Verena Schwarz
Universität Klagenfurt

In this talk we present a strong approximation result for the solution of jump-diffusion stochastic differential equations with discontinuous drift, possibly degenerate diffusion coefficient, and Lipschitz jump-diffusion. We construct a transformation-based jump-adapted quasi-Milstein scheme, which has convergence order 3/4 under additional piecewise smoothness assumptions to the drift and diffusion coefficient. To obtain this result we show that under slightly stronger assumptions on the coefficients the jump-adapted quasi-Milstein scheme also has convergence order 3/4. This is joint work with Pawel Przybylowicz and Michaela Szölgyenyi.

Random approximation of convex bodies in Hausdorff distance

Mathias Sonnleitner
Universität Passau

Approximating convex sets by the convex hull of random points is a subject firmly rooted in stochastic geometry. As a measure of the quality of approximation, the symmetric difference has attracted more attention than the Hausdorff distance. We focus on the latter and survey existing results. We fill some gaps in the case of a smooth boundary as well as for polygons in the plane, where we give asymptotic constants. This is based on joint work with Joscha Prochno, Carsten Schütt and Elisabeth Werner.

Predictable and Chaotic Representation Properties – Survey and New Developments (Invited Talk)

Alexander Steinecke
Montanuniversität Leoben

The best known example for a chaotic representation is the expansion of a random variable $\xi \in L^2(\Omega, \mathcal{F}^B, \mathbb{P})$ – measurable with respect to a σ-Algebra $\mathcal{F}^B$ generated by a Brownian motion B – into an infinite series. Summands of such a series depend on the approach and may take the form of multiple stochastic integrals (Wiener 1938, Itô 1956), iterated integrals (Itô 1956) or multiple Hermite polynomials (Cameron-Martin 1947). However, their starting term is always $\mathbb{E}[\xi]$. A similar chaotic representation can be established for the compensated Poisson process. The immediate question that arises was formulated by P.-A. Meyer in 1976: "Which martingales X satisfying $\langle X, X\rangle_t = t$ are those who admit a chaotic representation using iterated integrals with the integrator being the process X itself?"

After defining some key concepts in this still active area of research, this talk recalls and explains a key contribution given by Émery in 1989 : The connection between the chaotic representation property of a martingale X and solutions of the *structure equation*

$$[X, X]_t - t = \int_{]0,t]} (\alpha + \gamma X_{s-}) dX_s, \quad t \geq 0,$$

for some $\alpha, \gamma \in \mathbb{R}$, in particular for $\gamma \in [-2, 0]$.

We then focus on an important property for the existence of a representation: the density of the linear hull of the products $\left\{ \prod_{i=1}^n X_{t_i}, t_i \in [0, T] \right\}$ in $L^2(\Omega, \mathcal{F}^X, \mathbb{P})$. More generally, we present an investigation of the products $\left\{ \prod_{i=1}^n I_{m_i}(f_{m_i}) \right\}$, where $I_n(f_n)$ is an iterated integral of order n over a deterministic integrand f_n (in n variables). While the chaos expansion of a sum of two such stochastic integrals (due to the linearity) is clear, their product leads to quite involved formulae, which have been established in the Brownian (Lee and Shih 2014) and Poisson case (Agrawal et al. 2020, actually treating pure-jump Lévy processes). We want to go even further:

Among the class of Lévy processes, Brownian motion and the compensated Poisson process (up to real multiples) are the only ones that allow a chaotic representation. For other Lévy processes X, it is necessary to replace the integrator process X by a random measure M (taking into account the jumps of the process), or, one might go over to a chaotic representation involving not the process itself as integrator, but a whole integrator family $X^{(1)}, X^{(2)}, \ldots$. The construction of such a family using the structure equation was established by Di Tella and Engelbert in 2016. Using general product formulae of their article for iterated integrals, we establish a concise, explicit chaos expansion of products of multiple integrals in the Lévy setting. That is, we find a family of deterministic integrands $(g_k)_{k\geq 0}$ such that

$$I_{m_1}(f_{m_1}) \cdots I_{m_N}(f_{m_N}) = \sum_{k=0}^{\infty} I_k(g_k).$$

Based on joint work with Paolo Di Tella and Christel Geiss.

Existence, uniqueness, and approximation for jump-driven SDEs with discontinuous drift

Michaela Szölgyenyi
Universität Klagenfurt

In this talk we present an existence and uniqueness result of strong solutions to multi-dimensional jump-diffusion SDEs with discontinuous drift and general finite activity jumps. Jump-diffusion SDEs are used for example in models for applications in energy markets, where sudden movements of the energy price have to be captured. For a special scalar case, we study the strong convergence order of the Euler-Maruyama scheme and recover the optimal rate 1/2.

A branching particle system as a model of semi pushed fronts

Julie Tourniaire
IST Austria

We consider a system of particles performing a one-dimensional dyadic branching Brownian motion with space-dependent branching rate, negative drift $-\mu$ and killed upon reaching 0, starting with N particles. More precisely, particles branch at rate $\rho/2$ in the interval $[0, 1]$, for some $\rho > 1$, and at rate 1/2 in $(1, +\infty)$. The drift $\mu(\rho)$ is chosen in such a way that, heuristically, the system is critical in some sense: the number of particles stays roughly constant before it eventually dies out. This particle system can be seen as an analytically tractable model for fluctuating fronts, describing the internal mechanisms driving the invasion of a habitat by a cooperating population. Recent studies from Birzu, Hallatschek and Korolev suggest the existence of three classes of fluctuating fronts: pulled, semi-pushed and pushed fronts. Here, we rigorously verify and make precise this classification and focus on the semi-pushed regime. More precisely, we prove the existence of two critical values $1 < \rho_1 < \rho_2$ such that for all $\rho \in (\rho_1, \rho_2)$, there exists $\alpha(\rho) \in (1, 2)$ such that the rescaled number of particles in the system converges to an α-stable continuous-state branching process on the time scale $N^{\alpha-1}$ as N goes to infinity. This complements previous results from Berestycki, Berestycki and Schweinsberg for the case $\rho = 1$.

Qualitative properties of different numerical methods for the inhomogeneous geometric Brownian motion

Irene Tubikanec
Universität Klagenfurt

The inhomogeneous geometric Brownian motion (IGBM) is used to describe price fluctuations in mathematical finance or changes in the membrane voltage of a neuron in neuroscience. It is among the simplest stochastic differential equations for which there does not exist a known method of exact simulation. In this talk, we analyse and compare qualitative features of different numerical methods for the IGBM. The conditional and asymptotic mean and variance of the IGBM are known and the process can be characterised according to Feller?s boundary classification. We compare the frequently used Euler-Maruyama and Milstein methods, two Lie-Trotter and two Strang splitting schemes and two methods based on the ordinary differential equation (ODE) approach, namely the classical Wong-Zakai approximation and the recently proposed log-ODE scheme. First, we prove that, in contrast to the Euler-Maruyama and Milstein schemes, the splitting and ODE schemes preserve the boundary properties of the process, independently of the choice of the time discretisation step. Second, we derive closed-form expressions for the conditional and asymptotic means and variances of all considered schemes and analyse the resulting biases. While the Euler-Maruyama and Milstein schemes are the only methods which may have an asymptotically unbiased mean, the splitting and ODE schemes perform better in terms of variance preservation. The Strang schemes outperform the Lie-Trotter splittings, and the log-ODE scheme the classical ODE method. The mean and variance biases of the log-ODE scheme are very small for many relevant parameter settings. However, in some situations the two derived Strang splittings may be a better alternative, one of them requiring considerably less computational effort than the log-ODE method.

The heat equation and its applications to mixing time of walks on finite groups

Yuwen Wang
Universität Innsbruck

In this talk, we start with a quick introduction to the heat equation. Although usually the heat equation is studied on R^n, in fact, one can make sense

of the heat equation on many more spaces, like manifolds and ”sufficiently lattice like graphs.” Using these observations, Diaconis and Saloff-Coste ’97 show that simple random walks on ”lattice-like” graphs mix in time square of the diameter of the graph. Lastly, we will discuss recent work on generalizing these results to random walks with long-jumps for finite nilpotent groups. Joint work with Laurent Saloff-Coste.